

5.21

a) Total # of ways = 15^{15}
15 ways in which all passengers leave at same stop

$$15^{15} - 15$$

b)
$$\binom{15+15-1}{15} - 15$$

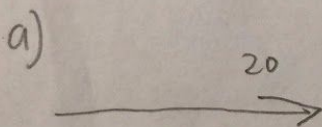
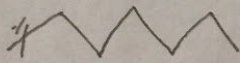
5.22

$$\binom{4+6-1}{4} - 6$$

3 colors

$$\left(\binom{4+6-1}{4} - 6 \right)^3$$

5.23

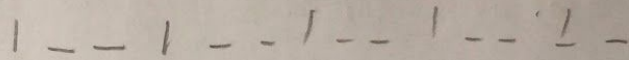


10 ups 10 downs

$$\binom{20}{10}$$

5.24

arrange red cards 1st $7 \times 2 = \frac{14}{6}$ ²⁰

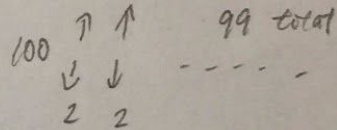


6 black cards left 9 spaces

$$81 \cdot \binom{6+9-1}{6}$$

5.25

a) 2^{99}



b) ?

b) ?

5.26

a) $10-3=7$

b) $\binom{10+4-1}{10} = \binom{13}{10} = 286$

$n(A) = n(B) = n(C) = n(D) = \binom{7+4-1}{2} = \binom{10}{7} = 120$ $10-3-3=4$

$n(A \cap B) = n(B \cap C) = n(A \cap C) = n(A \cap D) = n(B \cap D) = n(C \cap D) = \binom{4+4-1}{4} =$

$n(A \cap B \cap C) = n(A \cap B \cap D) = n(A \cap C \cap D) = n(B \cap C \cap D) = \binom{1+4-1}{3} = 35$

$n(A \cap B \cap C \cap D) = 0$

$n(A \cup B \cup C \cup D) = \binom{4}{1} \cdot 120 - \binom{4}{2} \cdot 35 + \binom{4}{3} \cdot 4 - \binom{4}{4} \cdot 0 =$

$480 - (6 \cdot 35) + 4 \cdot 4 - 0 = 286$

5.27

Circular permutation

$\frac{6!}{6}$

6 spaces used 12 cards. 8 left

$\binom{8+6-1}{8}$

arrange black cards

$20!$

$\frac{6!}{6} \cdot \binom{8+6-1}{8} \cdot 20!$

5.28

Complementary counting

$\frac{200}{8} \neq \text{integer}$

total # of ways

$\binom{200+3-1}{200} = 303$

$2a+b = 200$

$(0, 200) (1, 198) (2, 196) (3, 194) (4, 192) \dots$

a, b

$0-100 (100, 0)$

101 pairs

3 ways arrange

$101 \cdot 3 = 303$

5.29. $(2^a \cdot 5^x)$

$40 = 2^3 \cdot 5$ $(2^b \cdot 5^y)$

$2 \cdot 20 = (2^c \cdot 5^z)$

$2 \cdot 10 = 2^2 \cdot 5$

$a+b+c = 3$

$x+y+z = 1$

$\binom{3+3-1}{3} \cdot \binom{1+3-1}{1} = \binom{5}{3} \cdot \binom{3}{1}$

a) 30

b) $40 \cdot 1 \cdot 1$

$20 \cdot 2 \cdot 1$

$10 \cdot 4 \cdot 1$

$8 \cdot 5 \cdot 1$

$10 \cdot 2 \cdot 2$

$5 \cdot 4 \cdot 2$

5-30

same finger diff finger

$2! \cdot 4 \quad 4 \cdot 3$

$8 \quad + \quad 12$

$= 20$