

2.21

To get divisibility of 6,  
a number must be  
divisible by both 2 and 3.

$$b = 2 \cdot 3$$

b is a multiple of 2,  
so the number must be an  
even number.

b is also a multiple of 3,  
so the number must also  
be divisible by 3

Lastly, the LCM of 2 and  
3 is 6 so if any number  
is  $2/n$  and  $3/n$ , then  
 $2 \cdot 3/n \Rightarrow 6/n$

2.23

a)  $n = \overline{1a2a3a4a}$

$$9 \mid 1+2+3+4+4a =$$

$$9 \mid 10+4a$$

$$a = 2$$

b)  $n = \overline{1a2a3a4a}$

$$11 \mid -a - 4 + a - 3 + a - 2 + a - 1$$

$$11 \mid 4a - 10$$

$$32 - 10 = 22$$

$$a = 8$$

2.24

$$7 \mid 10a + b$$

2.22

$$a - b + c - d = 0 \text{ or } 11$$

$$n = \overline{abcd} = a \times 1000 + b \times 100 + c \times 10 + d$$

$$a(1001 - 1) + b(1 + 99) + c(11 - 1) + d$$

$$n = -a + b - c + d + 11 \times (91a + 9b + a)$$

$$11 \mid -a + b - c + d \text{ or}$$

$$11 \mid (a - b + c - d) \checkmark$$

2.25

$$11 \mid a - b + c - d$$

$$93 \ 57$$

$$97531$$

$$9 - 7 + 3 - 5 = 0$$

$$9735$$

2.26

$$\overline{3abcd}$$

$$-a + b - c + d = 11$$

$$11 \mid 3 - a + b - c + d$$

$$0 \ 1 \ 2 \ 4$$

$$3 - 0 + 1 - 2 + 4 = 6$$

$$30129$$

2.27

if  $n$  is odd, then either  $a$  or  $b$  is odd

Suppose  $a$  is odd

$$a = 2k + 1$$

$$b = 2l$$

$$a^2 = 4k^2 + 4k + 1$$

$$b^2 = 4l^2$$

$$a^2 + b^2 = 4k^2 + 4k + 1 + 4l^2 = 4m + 1$$

$$4k^2 + 4k + 4l^2 = 4m$$

$$m = k^2 + k + l^2$$

2.28

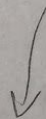
a)

2.29

2.30.  $44 \mid a357b$

$$11 \mid \overline{a357b}$$

$$4 \mid \overline{a357b}$$



$$4 \mid 7b \rightarrow 72 \text{ or } 76$$

$$\leftarrow b = 2 \text{ or } 6$$

$$a - 3 + 5 - 7 + 2 \quad \text{vs} \quad a - 3 + 5 - 7 + 6$$

$$a - 3 + 5 - 7 + 2 = 0 \text{ or } 11$$

$$a - 3 + 5 - 7 + 6 = 0 \text{ or } 11$$

$$a - 3 = 0 \text{ or } 11$$

$$a - 10 = 0 \text{ or } 11$$

$$a = 3 \text{ or } 14$$

$$a = 10 \text{ or } 21$$

$$a = 3 \quad b = 2$$