4.21

$31 + 28 + 31 + 30 + 31 = 151 + 1$ leap year

June 1st = 153rd day

$153 \equiv 6 \mod 5$

Jan 1, 2016 is Friday

5 6 0 1 2 3 :

Wednesday

4.22 d

4.22

a) If $b \equiv c \pmod m$ then $a \equiv c \mod m$

$\Downarrow$

$a - c = (a-b) - (b-c)$

$m | a-c$ since $m | a-b$ and $m | b-c$

b)

c) $m | (a-b)$ and $m | (c-d)$

k, l as integers

$km = a-b$ and $lm = c-d$

$a = km + b$ $\quad c = lm + d$

$ac - bd = (km+b)(lm+d) - bd =$

$klm^2 + blm + dkm + bd - bd =$

$m(klm + bl + dk)$

$m | ac - bd$

4.23

a)

$3^n \mod 10$ $\quad 7^n \mod 10$

$3^1 = 3$ $\quad 7^1 = 7$

$3^2 = 9$ $\quad 7^2 = 9$

$3^3 = 7$ $\quad 7^3 = 3$

$3^4 = 1$ $\quad 7^4 = 1$

$3^5 = 3$ $\quad 7^5 = 7$

every 4 $\quad$ every 4

$1234 \equiv 2 \mod 4$

$4321 \equiv 1 \mod 4$

$3^{1234} \equiv 3^2$

$7^{4321} \equiv 7^1$

$3^2 + 7^1 \equiv 9 + 7 \equiv 16 \equiv 6$

6

4.23

b) Yes

$23 \equiv -43 \mod 66$

$43^{101} + (-43)^{101} \equiv 0 \mod 66$

c) $n = 1, 2, 3, 4, 5, 6, 7$

$n^2 = 1, 4, 9, 16, 25, 36, 49$

$\equiv$

$1, 4, 2, 2, 4, 1, 0 \mod 7$

repeats

100 #s

$14 \cdot (1+4+2+2+4+1\ 10) + 1 + 4 = 5$

5 mod 7

4.25

a) $10^j - 1 = 99\ldots9$

$99\ldots9 = 0 \mod 3$

when there are j 9's

$10^j = 1 \mod 3$

$n = a_k 10^k + a_{k-1} 10^{k-1} \ldots a_1 10 + a_0 = a_k + a_{k-1} + \ldots a_1 + a_0 \mod 3$

b)

4.26

$52 + 522 + 5222 \ldots$

$522\ldots22$ (20 2's)

since it's even

$2 - 2 + 2 - 2 \ldots + 5 \equiv 5 \mod 11$

if odd

$2 - 2 + 2 - 2 + 2 \ldots + 2 - 5 \equiv -3 \mod 11$

$m + n = 5 + -3 = 2 \mod 11$

10 pairs $\Rightarrow 10 \times 2 = 20$

$20 \equiv 9 \mod 11$

4.24

$171 \mid 9$

$171 \div 9 = 19$

so # is div by 9

$9 + 2 + d + 1 + 1 = d + 13$

$d + 13 = \mod 9$

$d = 5$

4.27

$7^1 = 7$

$7^2 = 49 \equiv 11$

$7^3 = 343 \equiv 1$

$7^4 = 2401 \equiv 7$

$\Big\}\ 3$

$101/3 \equiv 2 \mod 3$

$7^{100} \equiv 7^2 \equiv 11 \mod 19$

4.28

$a_1 = 1 \equiv 1 \mod 4$

$a_2 = 1 \equiv 1 \mod 4$

$a_3 = 2(a_2) + a_1 = 1 + 2 = 3 \equiv 3 \mod 4$

$a_4 = 2(a_3) + a_2 = 7 \equiv 3 \mod 4$

$a_5 = 2(\overset{7}{a_4}) + \overset{3}{a_3} = 17 \equiv 1 \mod 4$

$a_6 = 2(\overset{17}{a_5}) + \overset{7}{a_4} = 41 \equiv 1 \mod 4$

$a_7 = 2(41) + 17 = 99 \equiv 3 \mod 4$

$a_8 = 2(99) + 41 = 239 \equiv 3 \mod 4$

$a_9 = 2(239) + 99 = 577 \equiv 1 \mod 4$

every 4 repeat

$2015 \equiv 3 \mod 4$

$a_3 \equiv 3 \mod 4$

4.30

4.29

if dive by 99
must div by 9 and 11

$b + 7 + 7 + 4 + a \equiv a + b + \cancel{18} \equiv 0 \mod 9$

$b - 7 + 7 - a + 4 \equiv b - a - 7 \equiv 0 \mod 11$

$b - a \equiv 7 \mod 11$

$b = 8$

$a = 1$

$b + a = 9$ 8 1 7 2

$b - a = 7$ 7 5