Math Challenge I-C: Finite Math

Answer Key

Areteem Institute

Chapter 1. Place Values, and Divisibility

Quick Reponse Questions:

1.11: Yes.

1.12: No.

1.13: C

1.14: Yes

1.15: No

1.16: 444

1.17: 12348

1.18: 4

1.19: 1243

1.20: 18

Practice Questions:

1.21: Check whether the number is divisible by 3 and by 5.

1.22: 13

1.23: Omitted

1.24: $a = 3$, $b = 2$

1.25: $(a,b) = (0,9),(1,8),(2,7),\ldots,(9,0)$

1.26: 10

1.27: No

1.28: 35222, 35828

1.29: 27

1.30: Omitted

Chapter 2. Primes, Factors, and Multiples

Quick Reponse Questions:

2.11: 53

2.12: 6

2.13: 11

2.14: 18

2.15: 15

2.16: 2002770

2.17: 70

2.18: 205800

2.19: 48

2.20: 45

Practice Questions:

2.21(a): $2016 = 2^5 \cdot 3^2 \cdot 7$, 36 factors

2.21(b): $30030 = 2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13$, 64 factors

2.22: 31

2.23(a): 5

2.23(b): 500

2.24: $B = 42, C = 45$

2.25: 60

2.26: 32, 12, 20, 28, 44, 18, 45

2.27: GCD: 2; LCM: 2400

2.28: 956

2.29: 3 or 9

2.30: $A = 100, B = 112$

Chapter 3. Modular Arithmetic

Quick Reponse Questions:

3.11: 8

3.12: 4

3.13: No

3.14: Yes

3.15: 68

3.16: 5

3.17: 8

3.18: 6

3.19: 8

3.20: 3

Practice Questions:

3.21: 10

3.22: 11

3.23: 91

3.24: Omitted

3.25(a): Omitted

3.25(b): Omitted

3.26: 0

3.27: 7

3.28: 10

3.29: Omitted

3.30: 5

Chapter 4. Sequences and Series

Quick Reponse Questions:

4.11: 35

4.12: B

4.13: 8

4.14: 1536

4.15: D

4.16: C

4.17: 5050

4.18: -8

4.19: 4

4.20: 2047

Practice Questions:

4.21(a): $3, 4, 2, 2, 4, 3, 4, 2, 2, 4$

4.21(b): $N = 5$

4.22(a): $a_n = -3 + 4n$

4.22(b): $a_n = 23 - 6n$

4.23(a): $a_n = 27 \cdot \left(\dfrac{2}{3}\right)^n$

4.23(b): $a_n = \dfrac{1}{64} \cdot (-4)^n$

4.24(a): Omitted

4.24(b): All the differences are multiples of the original sequence

4.25: Omitted

4.26(a): $\displaystyle\sum_{k=0}^{9} 4 \cdot 2^k$

4.26(b): $\displaystyle\sum_{k=0}^{15} 8 - 4k$

4.27: $\dfrac{n}{2}(a_0 + a_{n-1})$

4.28: $\dfrac{78}{99} = \dfrac{26}{33}$

4.29(a): $a \cdot \dfrac{r^n - 1}{r - 1}$

4.29(b): $\dfrac{1}{1 - r}$

4.30: 3

Chapter 5. Recursive Sequences

Quick Reponse Questions:

5.11: 180

5.12: 9

5.13: C

5.14: B

5.15: -100

5.16: 243

5.17: 3

5.18: D

5.19: 76

5.20: B

Practice Questions:

5.21: $0, 1, 5, 14, 30, \ldots$

5.22: $0, 1, 5, 12, 22, \ldots$

5.23: $a_0 = -3, a_{n+1} = a_n + 13$; $a_n = -3 + 13n$

5.24: $a_0 = 16, a_{n+1} = 1.5 \cdot a_n$; $a_n = 16 \cdot (1.5)^n$

5.25: Omitted

5.26: $H_n = (-1)^n$

5.27: Omitted

5.28: $a_n = 2 \cdot 2^n + 3$

5.29: $S_n = T_n$

5.30: It is a famous conjecture that the sequence always reaches 1.

Chapter 6. Counting Introduction

Quick Reponse Questions:

6.11: 1024

6.12: 2048

6.13: 44

6.14: C

6.15: 35

6.16: B

6.17: C

6.18: 126

6.19: 72

6.20: 6

Practice Questions:

6.21: 165

6.22(a): 10000

6.22(b): 5040

6.22(c): 4960

6.23(a): 120

6.23(b): 60

6.23(c): 10

6.24: 34650

6.25(a): 6

6.25(b): 15

6.26: 100

6.27: 252

6.28: 455

6.29: 511

6.30: 240

Chapter 7. Sets and Functions

Quick Reponse Questions:

7.11: D

7.12: C

7.13: B

7.14: 2

7.15: 7

7.16: C

7.17: B

7.18: D

7.19: A

7.20: 2

Practice Questions:

7.21(a): $n(A) = 10$, $n(B) = 4$

7.21(b): $n(A \cap B) = 2$ and $n(A^c \cap B) = 2$

7.22: 10

7.23: 24

7.24: Omitted

7.25: 44

7.26(a): Omitted

7.26(b): Omitted

7.27(a): 25, 32

7.27(b): 20

7.28: 30

7.29: $6^{20} - 3^{20}$.

7.30: 8

Chapter 8. Bijections and Stars and Bars

Quick Reponse Questions:

8.11: 84

8.12: 64

8.13: C

8.14: B

8.15: 28

8.16: 10

8.17: 2401

8.18: 210

8.19: 35

8.20: 840

Practice Questions:

8.21: 150

8.22: 5, note the numbers in the top row correspond to the positions of the U's in the path

8.23: Omitted

8.24(a): 32

8.24(b): Omitted

8.24(c): Omitted

8.25: Omitted

8.26(a): 10 stars, 4 bars

8.26(b): 1001

8.27: 32

8.28(a): $\binom{30+6-1}{30} = 324632$

8.28(b): $\binom{18+6-1}{18} = 33649$

8.28(c): $\binom{20+5-1}{20} = 10626$

8.29: $\binom{6+3-1}{6} - 3 = 25$

8.30: 720

Chapter 9. Probability

Quick Reponse Questions:

9.11: 67

9.12: 53

9.13: 4

9.14: 4

9.15: 0.25

9.16: 3

9.17: 0.4

9.18: 5

9.19: 21

9.20: 1

Practice Questions:

9.21(a): $\dfrac{1}{6}$

9.21(b): $\dfrac{5}{16}$

9.22: $\dfrac{121}{128}$

9.23(a): $\dfrac{1}{3}$

9.23(b): $\dfrac{1}{2}$

9.23(c): Omitted

9.24(a): $\dfrac{560}{1287}$

9.24(b): $\dfrac{2}{1287}$

9.25: $\dfrac{15}{256}$

9.26: Omitted

9.27: $\dfrac{11}{21}$

9.28: $P(A \cap B^c) = 0.5, P(A \cap B) = 0.2, P(A^c \cap B) = 0.3, P(A^c \cap B^c) = 0$

9.29(a): $\dfrac{1}{2}$

9.29(b): $\dfrac{1}{2}$

9.30: $\dfrac{5}{648}$