

Math Challenge II-A: Combinatorics

Answer Key

Areteem Institute

Chapter 1. Counting Fundamentals

Quick Reponse Questions:

1.11: B

1.16: D

1.12: C

1.17: B

1.13: B

1.18: B

1.14: A

1.19: C

1.15: C

1.20: D

Practice Questions:

1.21: 768.

1.25(a): $4 \cdot 5^9$

1.22(a): $8! = 40320$

1.25(b): $\binom{9}{2} \cdot 4^8$

1.22(b): $\frac{8!}{5!} = 8 \cdot 7 \cdot 6 = 336.$

1.26: 4.

1.22(c): $\binom{8}{4} = \frac{8!}{4! \cdot 4!} = 70.$

1.27: 152.

1.23(a): $\frac{8!}{3!} = \binom{8}{3} \cdot 5!.$

1.28: $26 \cdot 25^6 - 21 \cdot 20 \cdot 25^5 = 5 \cdot 25^5 + 21 \cdot 5 \cdot 25^5.$

1.23(b): $2! \cdot 6!.$

1.29: $6 \cdot 5 \cdot 6 \cdot 5 = 900.$

1.24: 10

1.30(a): $5 \cdot 8^6 - 5 = 1310715.$

1.30(b): $5 \cdot 8^2 \cdot \frac{8!}{4!}$

Chapter 2. Counting Methods I

Quick Reponse Questions:

2.11: A

2.12: C

2.13: A

2.14: C

2.15: D

2.16: 14400

2.17: 462

2.18: 1152

2.19: C

2.20: C

Practice Questions:

2.21(a): 342

2.21(b): $7 \cdot 6 + 12 \cdot 18 = 258$.

2.22: $\binom{10}{3} \cdot \binom{10}{2} \cdot \binom{10}{1} \cdot \binom{7}{3} \cdot \binom{8}{2} \cdot \binom{9}{1}$

2.23: $4! \cdot (2!)^4$

2.24: $10 \cdot 9 \cdot 8 \cdot 7 = 5040$

2.25: $315 - 70 = 245$.

2.26: $\left(\binom{3}{2} + \binom{9}{6} \right) \cdot \binom{120}{40}$.

2.27: $\frac{\binom{20}{5} \cdot \binom{15}{5} \cdot \binom{10}{5} \cdot \binom{5}{5}}{2! \cdot 2!}$

2.28: $5! \cdot 6! \cdot 6! \cdot 7 \cdot 6 = 2612736000$.

2.29: $\frac{6!}{2!} \cdot 8 \cdot 7 = 20160$.

2.30: $3! \cdot (2! \cdot 2! \cdot 2!) = 48$.

Chapter 3. Counting Methods II

Quick Reponse Questions:

3.11: 6

3.12: 286

3.13: 64

3.14: 26

3.15: 3840

3.16: 240

3.17: 84

3.18: 1048576

3.19: 5040

3.20: 9

Practice Questions:

3.21: $2! \cdot \frac{7!}{7} \cdot 8 = 11520.$

3.22: 128.

3.23: 15

3.24(a): $\binom{12+6-1}{12} = 6188.$

3.24(b): $\binom{6+6-1}{6} = 462.$

3.25: $\binom{9+3-1}{9} + \binom{8+3-1}{8} + \binom{7+3-1}{7} = 136.$

3.26: $\binom{4+3-1}{4}^3 = 3375.$

3.27: 3.

3.28: $\binom{30}{10} \cdot 5^{20}$

3.29: $\frac{5!}{5} \cdot \binom{9}{5} \cdot 15!$

3.30(a): $\binom{5+10-1}{5} = 2002.$

3.30(b): $\binom{10}{5}.$

Chapter 4. Sets and Inclusion-Exclusion

Quick Reponse Questions:

4.11: C

4.16: 46

4.12: A

4.17: 24

4.13: D

4.18: 11520

4.14: 7

4.19: 1001

4.15: 74

4.20: 768

Practice Questions:

4.21: Omitted

4.27: 366

4.22: Omitted

4.28: 2

4.23: 46

4.29: 44

4.24: 11

4.30(a): $3^5 - 3 \cdot 2^5 + 3 = 150.$

4.25: 5796

4.30(b): $\binom{3}{2} \cdot \frac{5!}{1! \cdot 1! \cdot 3!} + \binom{3}{2} \cdot \frac{5!}{2! \cdot 2! \cdot 1!} = 150.$

4.26: $6^{40} - 3^{40}$

Chapter 5. Advanced Counting

Quick Reponse Questions:

5.11: 625

5.12: A

5.13: 39

5.14: C

5.15: 64

5.16: 144

5.17: 91

5.18: 78

5.19: D

5.20: 36

Practice Questions:

5.21(a): $15^{15} - 15$

5.21(b): $\binom{15+15-1}{15} - 15$

5.22: $\left(\binom{4+6-1}{4} - 6\right)^3$

5.23(a): $\binom{20}{10} = 184756$

5.23(b): $\binom{10}{5}^2 = 63504.$

5.24: $8! \cdot \binom{6+9-1}{6}$

5.25(a): 2^{99}

5.25(b): 100

5.26(a): 286

5.26(b): Omitted

5.27: $\frac{6!}{6} \cdot \binom{8+6-1}{8} \cdot 20!$

5.28: 19998

5.29(a): $\binom{3+3-1}{3} \cdot \binom{1+3-1}{1} = 30$

5.29(b): 6

5.30: 20

Chapter 6. Combinatorial Identities

Quick Reponse Questions:

6.11: 1287

6.16: 84

6.12: 60

6.17: 0

6.13: 126

6.18: 1024

6.14: C

6.19: 250

6.15: 17

6.20: 2520

Practice Questions:

6.21(a): Omitted

6.26: 625

6.21(b): Omitted

6.27: 1024

6.22: Omitted

6.28: $90x^2y^2 + 180x^2y^2z + 90x^2y^2z^2$.

6.23: Omitted

6.29: 125.

6.24: Omitted

6.30: Omitted

6.25: Omitted

Chapter 7. Recurrence Relations

Quick Reponse Questions:

7.11: 1287

7.16: 3

7.12: 40095

7.17: D

7.13: 6930

7.18: 125

7.14: 2187

7.19: 1716

7.15: 1

7.20: 9

Practice Questions:

7.21: $a_0 = a_1 = 1, a_n = 2^{n-1}$.

7.26(b): Omitted

7.22: $a_1 = 1, a_{n+1} = a_n + (2n+1)^2$.

7.27: $\binom{7}{3} - \binom{5}{3} = 25$.

7.23: 24165120

7.28: 13

7.24: Write $a \in A$ in binary. If the k -th position in a is a 1, then include k in the set.

7.29(a): Omitted

7.25: Omitted

7.29(b): Omitted

7.26(a): $a_n = 2^n - 2$.

7.30: Omitted

Chapter 8. Probability

Quick Reponse Questions:

8.11: 462

8.16: 21

8.12: -3240

8.17: 0.3

8.13: 105

8.18: 0.4

8.14: B

8.19: 3

8.15: 3

8.20: 32

Practice Questions:

8.21: $\frac{63}{125}$.

8.26: 0.2

8.22(a): $\frac{219}{256}$.

8.27(a): $\binom{13}{1}\binom{12}{1}\binom{4}{4}\binom{4}{1} / \binom{52}{5}$

8.22(b): $\frac{7}{256}$.

8.27(b): $\binom{13}{1}\binom{12}{3}\binom{4}{2}\binom{4}{1}^3 / \binom{52}{5}$.

8.23: $\frac{3}{5}$.

8.28: $\frac{2\pi}{6\sqrt{3}} \approx .605$.

8.24: $\frac{15}{32}$.

8.29: $\frac{35}{68}$

8.25(a): $\frac{1}{15}$.

8.30: $\frac{5}{6}$

8.25(b): $\frac{2}{3}$.

Chapter 9. Advanced Probability

Quick Reponse Questions:

9.11: 0.16

9.16: D

9.12: 3901

9.17: 40

9.13: 125

9.18: B

9.14: 131

9.19: No

9.15: 307

9.20: Yes

Practice Questions:

9.21: $P(A|B) = \frac{2}{11}$ and $P(B|A) = \frac{2}{15}$.

9.25(a): $\frac{11}{50}$

9.22: $5/9$.

9.25(b): $\frac{22}{425}$

9.23(a): $(3!) \cdot \frac{4}{9} \cdot \frac{3}{9} \cdot \frac{2}{9}$.

9.26: $\frac{1}{2} \cdot \frac{5}{36} + \frac{1}{3} \cdot \frac{21}{216} + \frac{1}{6} \cdot \frac{35}{1296}$.

9.23(b): $\binom{3}{2} \cdot \left(\frac{4}{9}\right)^2 \cdot \left(\frac{5}{9}\right)$.

9.27: $5/11$

9.24: $P(A|C)$

9.28: No.

9.29: $\frac{1219}{15625}$

9.30: $1/3$