

Math Challenge II-A: Number Theory

Answer Key

Areteem Institute

Chapter 1. Place Values and Number Bases

Quick Reponse Questions:

1.11: 2479

1.16: 1010100

1.12: 127

1.17: 612

1.13: 531

1.18: 540

1.14: 492

1.19: 1043

1.15: 243

1.20: 0.011

Practice Questions:

1.21(a): 109

1.25: Yes

1.21(b): 173

1.26: Yes

1.22(a): 1102

1.27(a): Yes, base 7

1.22(b): BE_{16}

1.27(b): No

1.23(a): A3

1.28: 43

1.23(b): 1001010101011

1.29: Omitted

1.24(a): 212220_3

1.30: 10

1.24(b): $200,000_{16}$

Chapter 2. Divisibility

Quick Reponse Questions:

2.11: No

2.16: 12348

2.12: Yes

2.17: 4

2.13: No

2.18: 1243

2.14: No

2.19: 18

2.15: 882

2.20: 0

Practice Questions:

2.21: Omitted

2.26: 30129

2.22: Omitted

2.27: Omitted

2.23(a): $a = 2$

2.28(a): Omitted

2.23(b): $a = 8$

2.28(b): Omitted

2.24: Omitted

2.29: $8 \cdot 10^2 = 800$

2.25: 9735

2.30: $a = 3, b = 2$

Chapter 3. Prime Numbers

Quick Reponse Questions:

3.11: 17

3.16: 2002770

3.12: 3

3.17: 70

3.13: 12

3.18: 205800

3.14: 18

3.19: 45

3.15: 15

3.20: 48

Practice Questions:

3.21: Omitted

3.26(b): 19

3.22: 176400

3.27: 32, 12, 20, 28, 44, 18, 45

3.23: 60^{90}

3.28: $p = 3, q = 7$

3.24: Omitted

3.29: 69

3.25: 1

3.30: Omitted

3.26(a): Omitted

Chapter 4. Modular Arithmetic

Quick Reponse Questions:

4.11: 8

4.16: 8

4.12: 4

4.17: 4

4.13: No

4.18: 6

4.14: Yes

4.19: 3

4.15: 5

4.20: 0

Practice Questions:

4.21: Thursday

4.24: 4

4.22(a): Omitted

4.25(a): Omitted

4.22(b): Omitted

4.25(b): Omitted

4.22(c): Omitted

4.26: 5

4.22(d): Omitted

4.27: 4

4.23(a): 4

4.28: 1

4.23(b): Yes

4.29: $(a, b) = (0, 9), (1, 8), (2, 7), \dots, (9, 0)$

4.23(c): 0

4.30: 27

Chapter 5. Problem Solving in Mod. Arithmetic

Quick Reponse Questions:

5.11: 48

5.16: 503

5.12: 20

5.17: No

5.13: 5

5.18: Yes

5.14: 43

5.19: 12

5.15: 500

5.20: 6

Practice Questions:

5.21(a): $n \cdot 5 = 0, 5, 2, 7, 4, 1, 6, 3;$
 $n \cdot 6 = 0, 6, 4, 2, 0, 6, 4, 2.$

5.24: 0, 1, 8

5.21(b): 1, 3, 5, 7; $1^2 \equiv 3^2 \equiv 5^2 \equiv$
 $7^2 \equiv 1 \pmod{8}$

5.25: Omitted

5.26: Omitted

5.22: Omitted

5.27: Omitted

5.23(a): 1, 2, 4, 5, 7, 8

5.28: No

5.23(b): $2 \cdot 5 \equiv 4 \cdot 7 \equiv 1 \pmod{9},$
 $1^2 \equiv 8^2 \equiv 1 \pmod{9}.$

5.29: Yes

5.30: No

Chapter 6. Basic Theorems in Mod. Arith.

Quick Reponse Questions:

6.11: 7

6.16: 9

6.12: 3

6.17: 18

6.13: 6

6.18: 2

6.14: Yes

6.19: 1

6.15: No

6.20: 100

Practice Questions:

6.21: Omitted

6.26(b): 6.

6.22: Omitted

6.27: 9.

6.23(a): Omitted

6.28: 5.

6.23(b): Omitted

6.29(a): Answers may vary, $n = 9$ works.

6.23(c): Omitted

6.29(b): Answers may vary, $p = 3$ works.

6.24: Omitted

6.25: 10.

6.30: Omitted

6.26(a): 2.

Chapter 7. Advanced Theorems in Mod. Arith.

Quick Reponse Questions:

7.11: 240

7.16: 97

7.12: 560

7.17: 75

7.13: 8

7.18: 23

7.14: 17

7.19: No

7.15: 17

7.20: 103

Practice Questions:

7.21: $\phi(42) = 12$

7.25: $x = 40 + 77k$ for all k

7.22(a): $\phi(p^k) = p^k - p^{k-1}$

7.26(a): 1

7.22(b): $\phi(p^k \cdot q^l) = (p^k - p^{k-1})(q^l - q^{l-1})$.

7.26(b): 1

7.22(c): 160

7.26(c): 16

7.23(a): 29

7.27: $x = 705 + 900k$ for all k

7.23(b): 121

7.28: Omitted

7.24: 8

7.29: 2

7.30: Omitted

Chapter 8. Diophantine Equations

Quick Reponse Questions:

8.11: Yes

8.16: 2

8.12: No

8.17: Yes

8.13: 8

8.18: No

8.14: 3

8.19: Yes

8.15: 5

8.20: No

Practice Questions:

8.21: Omitted

8.26: $(x, y) = (3, 12), (2, 4)$

8.22: $(17, 144, 145), (15, 8, 17)$

8.27: Omitted

8.23: 60

8.28(a): $m = 0, n = 3$ and $m = 2, n = 4$

8.24(a): C must be a multiple of $\gcd(A, B)$

8.28(b): $m = 1, n = 3$ and $m = 3, n = 5$

8.24(b): Omitted

8.29: Omitted

8.25(a): Omitted

8.30: $(5, 2, 2), (2, 5, 5), (2, 3, 11)$

8.25(b): $x = 1, y = 2; x = -3, y = 4.$

Chapter 9. Floor Function

Quick Reponse Questions:

9.11: -1

9.16: No

9.12: 1

9.17: 5

9.13: 2

9.18: 4

9.14: 3

9.19: 4.5

9.15: 14

9.20: 6.83

Practice Questions:

9.21: Omitted

9.26(b): $\lfloor 2x \rfloor = 2\lfloor x \rfloor + 1$

9.22: 334

9.27: $\frac{9}{11} = \frac{1}{2} + \frac{1}{4} + \frac{1}{15} + \frac{1}{660}$

9.23: $2 - 3\sqrt{2} + 3\sqrt{3}$

9.28: $\frac{5}{91} = \frac{1}{28} + \frac{1}{52}$

9.24: $x = 0$

9.29: $(\sqrt{5} + 1)/2$

9.25: $-1/2, -5/2, -9/2$

9.30: $0, \sqrt{2}, \sqrt{3}, 2$

9.26(a): $n \leq x < n + \frac{1}{2}$ for n an integer