

# **Math Challenge II-B: Combinatorics**

**Answer Key**

**Areteem Institute**

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## Chapter 1. Counting Fundamentals

### Quick Reponse Questions:

1.11: B

1.16: D

1.12: C

1.17: B

1.13: B

1.18: B

1.14: A

1.19: C

1.15: C

1.20: D

### Practice Questions:

1.21: 768.

1.25(a):  $4 \cdot 5^9$

1.22(a):  $8! = 40320$

1.25(b):  $\binom{9}{2} \cdot 4^8$

1.22(b):  $\frac{8!}{5!} = 8 \cdot 7 \cdot 6 = 336.$

1.26: 4.

1.22(c):  $\binom{8}{4} = \frac{8!}{4! \cdot 4!} = 70.$

1.27: 152.

1.23(a):  $\frac{8!}{3!} = \binom{8}{3} \cdot 5!.$

1.28:  $26 \cdot 25^6 - 21 \cdot 20 \cdot 25^5 = 5 \cdot 25^5 + 21 \cdot 5 \cdot 25^5.$

1.23(b):  $2! \cdot 6!.$

1.29:  $6 \cdot 5 \cdot 6 \cdot 5 = 900.$

1.24: 10

1.30(a):  $5 \cdot 8^6 - 5 = 1310715.$

1.30(b):  $5 \cdot 8^2 \cdot \frac{8!}{4!}$

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## Chapter 2. Counting Methods I

### Quick Reponse Questions:

**2.11:** A

**2.16:** 14400

**2.12:** C

**2.17:** 462

**2.13:** A

**2.18:** 1152

**2.14:** C

**2.19:** C

**2.15:** D

**2.20:** C

### Practice Questions:

**2.21(a):** 342

**2.26:**  $\left( \binom{3}{2} + \binom{9}{6} \right) \cdot \binom{120}{40}$ .

**2.21(b):**  $7 \cdot 6 + 12 \cdot 18 = 258$ .

**2.22:**  $\binom{10}{3} \cdot \binom{10}{2} \cdot \binom{10}{1} \cdot \binom{7}{3} \cdot \binom{8}{2} \cdot \binom{9}{1}$

**2.27:**  $\frac{\binom{20}{5} \cdot \binom{15}{5} \cdot \binom{10}{5} \cdot \binom{5}{5}}{2! \cdot 2!}$

**2.23:**  $4! \cdot (2!)^4$

**2.28:**  $5! \cdot 6! \cdot 6! \cdot 7 \cdot 6 = 2612736000$ .

**2.24:**  $10 \cdot 9 \cdot 8 \cdot 7 = 5040$ .

**2.29:**  $\frac{6!}{2!} \cdot 8 \cdot 7 = 20160$ .

**2.25:**  $315 - 70 = 245$ .

**2.30:**  $3! \cdot (2! \cdot 2! \cdot 2!) = 48$ .

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## Chapter 3. Counting Methods II

### Quick Reponse Questions:

**3.11:** 6

**3.12:** 286

**3.13:** 64

**3.14:** 26

**3.15:** 3840

**3.16:** 240

**3.17:** 84

**3.18:** 1048576

**3.19:** 5040

**3.20:** 9

### Practice Questions:

**3.21:**  $2! \cdot \frac{7!}{7} \cdot 8 = 11520.$

**3.22:** 128.

**3.23:** 15

**3.24(a):**  $\binom{12+6-1}{12} = 6188.$

**3.24(b):**  $\binom{6+6-1}{6} = 462.$

**3.25:**  $\binom{9+3-1}{9} + \binom{8+3-1}{8} + \binom{7+3-1}{7} = 136.$

**3.26:**  $\binom{4+3-1}{4}^3 = 3375.$

**3.27:** 3.

**3.28:**  $\binom{30}{10} \cdot 5^{20}$

**3.29:**  $\frac{5!}{5} \cdot \binom{9}{5} \cdot 15!$

**3.30(a):**  $\binom{5+10-1}{5} = 2002.$

**3.30(b):**  $\binom{10}{5}.$

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## Chapter 4. Sets and Inclusion-Exclusion

### Quick Reponse Questions:

**4.11:** C

**4.16:** 46

**4.12:** A

**4.17:** 24

**4.13:** D

**4.18:** 11520

**4.14:** 7

**4.19:** 1001

**4.15:** 74

**4.20:** 768

### Practice Questions:

**4.21:** Omitted

**4.27:** 366

**4.22:** Omitted

**4.28:** 2

**4.23:** 46

**4.29:** 44

**4.24:** 11

**4.30(a):**  $3^5 - 3 \cdot 2^5 + 3 = 150$ .

**4.25:** 5796

**4.30(b):**  $\binom{3}{2} \cdot \frac{5!}{1! \cdot 1! \cdot 3!} + \binom{3}{2} \cdot \frac{5!}{2! \cdot 2! \cdot 1!} = 150$ .

**4.26:**  $6^{40} - 3^{40}$

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## Chapter 5. Advanced Counting

### Quick Reponse Questions:

**5.11:** 625

**5.16:** 144

**5.12:** A

**5.17:** 91

**5.13:** 39

**5.18:** 78

**5.14:** C

**5.19:** D

**5.15:** 64

**5.20:** 36

### Practice Questions:

**5.21(a):**  $15^{15} - 15$

**5.25(b):** 100

**5.21(b):**  $\binom{15+15-1}{15} - 15$

**5.26(a):** 286

**5.22:**  $\left(\binom{4+6-1}{4} - 6\right)^3$

**5.26(b):** Omitted

**5.27:**  $\frac{6!}{6} \cdot \binom{8+6-1}{8} \cdot 20!$

**5.23(a):**  $\binom{20}{10} = 184756$

**5.28:** 19998

**5.23(b):**  $\binom{10}{5}^2 = 63504.$

**5.29(a):**  $\binom{3+3-1}{3}.$

$\binom{1+3-1}{1} = 30$

**5.24:**  $8! \cdot \binom{6+9-1}{6}$

**5.29(b):** 6

**5.25(a):**  $2^{99}$

**5.30:** 20

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## Chapter 6. Combinatorial Identities

### Quick Reponse Questions:

**6.11:** 1287

**6.16:** 84

**6.12:** 60

**6.17:** 0

**6.13:** 126

**6.18:** 1024

**6.14:** C

**6.19:** 250

**6.15:** 17

**6.20:** 2520

### Practice Questions:

**6.21(a):** Omitted

**6.26:**  $3^n$

**6.21(b):** Omitted

**6.27:**  $n2^{n-1}$

**6.22:** Omitted

**6.28:**  $\frac{9!}{4!2!3!}x^4y^2z^3, 4 \cdot \frac{9!}{4!2!2!}x^4y^2z^2,$   
 $4^2 \cdot \frac{9!}{4!2!2!}x^4y^2z, 4^3 \cdot \frac{9!}{4!2!3!}x^4y^2z$

**6.23:** Omitted

**6.24:** Omitted

**6.29:** 52800

**6.25:** Omitted

**6.30:** Omitted

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## Chapter 7. Recurrence Relations

### Quick Reponse Questions:

**7.11:** 1287

**7.16:** 3

**7.12:** 40095

**7.17:** D

**7.13:** 6930

**7.18:** 125

**7.14:** 2187

**7.19:** 1716

**7.15:** 1

**7.20:** 9

### Practice Questions:

**7.21:**  $a_0 = a_1 = 1, a_n = 2^{n-1}$ .

**7.26(b):** Omitted

**7.22:**  $a_1 = 1, a_{n+1} = a_n + (2n+1)^2$ .

**7.27:**  $\binom{7}{3} - \binom{5}{3} = 25$ .

**7.23:** 24165120

**7.28:** 13

**7.24:** Write  $a \in A$  in binary. If the  $k$ -th position in  $a$  is a 1, then include  $k$  in the set.

**7.29(a):** Omitted

**7.25:** Omitted

**7.29(b):** Omitted

**7.26(a):**  $a_n = 2^n - 2$ .

**7.30:** Omitted



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## Chapter 8. Probability

### Quick Reponse Questions:

**8.11:** 462

**8.16:** 21

**8.12:** -3240

**8.17:** 0.3

**8.13:** 105

**8.18:** 0.4

**8.14:** B

**8.19:** 3

**8.15:** 3

**8.20:** 32

### Practice Questions:

**8.21:**  $\frac{63}{125}$ .

**8.26:** 0.2

**8.22(a):**  $\frac{219}{256}$ .

**8.27(a):**  $\binom{13}{1} \binom{12}{1} \binom{4}{4} \binom{4}{1} / \binom{52}{5}$

**8.22(b):**  $\frac{7}{256}$ .

**8.27(b):**  $\binom{13}{1} \binom{12}{3} \binom{4}{2} \binom{4}{1}^3 / \binom{52}{5}$ .

**8.23:**  $\frac{3}{5}$ .

**8.28:**  $\frac{2\pi}{6\sqrt{3}} \approx .605$ .

**8.24:**  $\frac{15}{32}$ .

**8.29:**  $\frac{35}{68}$

**8.25(a):**  $\frac{1}{15}$ .

**8.30:**  $\frac{5}{6}$

**8.25(b):**  $\frac{2}{3}$ .

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## Chapter 9. Advanced Probability

### Quick Reponse Questions:

**9.11:** 0.16

**9.16:** 40

**9.12:** 3901

**9.17:** B

**9.13:** 131

**9.18:** No

**9.14:** 307

**9.19:** Yes

**9.15:** D

**9.20:** 6

### Practice Questions:

**9.21:**  $P(A|B) = \frac{2}{11}$  and  $P(B|A) = \frac{2}{15}$

**9.25(a):**  $\frac{11}{50}$

**9.22:**  $\frac{5}{9}$

**9.25(b):**  $\frac{22}{425}$

**9.23(a):**  $\frac{16}{81}$

**9.26:**  $\frac{1}{2} \cdot \frac{5}{36} + \frac{1}{3} \cdot \frac{21}{216} + \frac{1}{6} \cdot \frac{35}{1296}$

**9.23(b):**  $\frac{80}{243}$

**9.27:** 96%

**9.24:**  $P(A|C)$

**9.28:**  $\frac{5}{11}$

**9.29:** No.

**9.30:**  $\frac{1}{3}$