Math Challenge II-B: Number Theory

Answer Key

Areteem Institute

Chapter 1. Place Values and Number Bases

Quick Reponse Questions:

1.11: 2479

1.12: 127

1.13: 531

1.14: 492

1.15: 243

1.16: 1010100

1.17: 612

1.18: 540

1.19: 1043

1.20: 0.011

Practice Questions:

1.21(a): 109

1.21(b): 173

1.22(a): 1102

1.22(b): BE_{16}

1.23(a): $A3$

1.23(b): 1001010101011

1.24(a): 212220_3

1.24(b): $200,000_{16}$

1.25: Yes

1.26: Yes

1.27(a): Yes, base 7

1.27(b): No

1.28: 43

1.29: Omitted

1.30: 10

Chapter 2. Divisibility

Quick Reponse Questions:

2.11: No

2.12: Yes

2.13: No

2.14: No

2.15: 882

2.16: 12348

2.17: 4

2.18: 1243

2.19: 18

2.20: 0

Practice Questions:

2.21: Omitted

2.22: Omitted

2.23(a): $a = 2$

2.23(b): $a = 8$

2.24: Omitted

2.25: 9735

2.26: 30129

2.27: Omitted

2.28(a): Omitted

2.28(b): Omitted

2.29: 9000

2.30: $a = 3, b = 2$

Chapter 3. Prime Numbers

Quick Reponse Questions:

3.11: 17

3.12: 3

3.13: 12

3.14: 18

3.15: 15

3.16: 2002770

3.17: 70

3.18: 205800

3.19: 45

3.20: 48

Practice Questions:

3.21: Omitted

3.22: 176400

3.23: 120^{189}

3.24: Omitted

3.25: 1

3.26(a): Omitted

3.26(b): 19

3.27: 32, 12, 20, 28, 44, 18, 45

3.28: $(1,1)$, $(3,3)$

3.29: 69

3.30: Omitted

Chapter 4. Modular Arithmetic

Quick Reponse Questions:

4.11: 8

4.12: 4

4.13: No

4.14: Yes

4.15: 5

4.16: 8

4.17: 4

4.18: 6

4.19: 3

4.20: 0

Practice Questions:

4.21: Wednesday

4.22(a): Omitted

4.22(b): Omitted

4.22(c): Omitted

4.22(d): Omitted

4.23(a): 6

4.23(b): Yes

4.23(c): 1

4.24: 5

4.25(a): Omitted

4.25(b): Omitted

4.26: 9

4.27: 11

4.28: 3

4.29: $a = 1, b = 8$

4.30: 89

Chapter 5. Problem Solving in Mod. Arith.

Quick Reponse Questions:

5.11: 48

5.12: 20

5.13: 5

5.14: 43

5.15: 500

5.16: 503

5.17: No

5.18: Yes

5.19: 12

5.20: 6

Practice Questions:

5.21(a): We have $n \cdot 5 = 0,5,10,3,8,1,6,11,4,9,2,7$ and $n \cdot 8 = 0,8,4,0,8,4,0,8,4,0,8,4$ for $n = 0,\ldots,11$.

5.21(b): $1,5,7,11$; $1^2 \equiv 5^2 \equiv 7^2 \equiv 11^2 \equiv 1 \pmod{12}$.

5.22: Omitted

5.23(a): 1, 2, 4, 7, 8, 11, 13, 14

5.23(b): $2 \cdot 8 \equiv 7 \cdot 13 \equiv 1 \pmod{15}$, $1^2 \equiv 4^2 \equiv 11^2 \equiv 14^2 \equiv 1 \pmod{15}$.

5.24: 0, 1, 8

5.25: Omitted

5.26: Omitted

5.27: Omitted

5.28: No

5.29: Yes

5.30: No

Chapter 6. Fundamental Theorems in Mod. Arith.

Quick Reponse Questions:

6.11: 7

6.12: 3

6.13: 6

6.14: Yes

6.15: No

6.16: 9

6.17: 18

6.18: 2

6.19: 1

6.20: 100

Practice Questions:

6.21: Omitted

6.22: Omitted

6.23(a): Omitted

6.23(b): Omitted

6.23(c): Omitted

6.24: Omitted

6.25: 10

6.26(a): 19

6.26(b): 4

6.27: 9

6.28: 5

6.29(a): Answers may vary, $n = 9$ works.

6.29(b): Answers may vary, $p = 3$ works.

6.30: Omitted

Chapter 7. Advanced Theorems in Mod. Arith.

Quick Reponse Questions:

7.11: 240

7.12: 560

7.13: 8

7.14: 17

7.15: 17

7.16: 97

7.17: 75

7.18: 23

7.19: No

7.20: 103

Practice Questions:

7.21: $\phi(60) = 16$

7.22(a): $\phi(p^k) = p^k - p^{k-1}$

7.22(b): $\phi(p^k \cdot q^l) = (p^k - p^{k-1})(q^l - q^{l-1})$.

7.22(c): 160

7.23(a): 29

7.23(b): 121

7.24: 7

7.25: $x = 48 + 55k$ for integers k

7.26(a): 1

7.26(b): 4

7.26(c): 8

7.27: $x = 785 + 1122k$ for all k

7.28: Omitted

7.29: 1

7.30: Omitted

Chapter 8. Diophantine Equations

Quick Reponse Questions:

8.11: Yes

8.12: No

8.13: 8

8.14: 3

8.15: 5

8.16: 2

8.17: Yes

8.18: No

8.19: Yes

8.20: No

Practice Questions:

8.21: $(3,4,5)$, $(5,12,13)$, $(8,15,17)$, $(7,24,25)$, $(20,21,29)$, $(12,35,37)$, $(9,40,41)$

8.22: $(20,21,29)$; $(29,420,421)$

8.23: 60

8.24(a): C must be a multiple of $\gcd(A,B)$

8.24(b): Omitted

8.25(a): Omitted

8.25(b): $x = \pm 4, y = 3$

8.26: $(x,y) = (3,12), (2,4)$

8.27: Omitted

8.28(a): $m = 0, n = 3$ and $m = 2, n = 4$

8.28(b): $m = 1, n = 3$ and $m = 3, n = 5$

8.29: Omitted

8.30: $(5,2,2)$, $(2,5,5)$, $(2,3,11)$

Chapter 9. Floor Function

Quick Reponse Questions:

9.11: -1

9.12: 1

9.13: 2

9.14: 3

9.15: 14

9.16: No

9.17: 5

9.18: 4

9.19: 4.5

9.20: 6.83

Practice Questions:

9.21: Omitted

9.22: 1004

9.23: $2-3\sqrt{2}+3\sqrt{3}$

9.24: $x=0$

9.25: $13/8, 3, 35/8$

9.26(a): x an integer.

9.26(b): $\lfloor -x \rfloor = -\lfloor x \rfloor - 1$

9.27: $\frac{9}{11} = \frac{1}{2}+\frac{1}{4}+\frac{1}{15}+\frac{1}{660}$

9.28: $\frac{5}{91} = \frac{1}{28}+\frac{1}{52}$

9.29: $(\sqrt{5}+1)/2$

9.30: $0, \sqrt{7}, 2\sqrt{2}, 3$