

# **Math Challenge III: Algebra**

**Answer Key**

**Areteem Institute**

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## Chapter 1. Review of Logarithm

### Practice Questions:

**1.16:** 1

**1.17:** 10

**1.18:** 125

**1.19:** 31

**1.20:** 2

**1.21:**  $\frac{1}{2}$

**1.22:** 2 or 5.

**1.23:** 10000

**1.24:** 210

**1.25:** 128

**1.26:** 4000000

**1.27:** 1 or 100

**1.28:**  $1 < a < \frac{1 + \sqrt{5}}{2}$

**1.29:**  $(x, y, z) = (1, 5, 1)$  or  $(100, 20, 100)$

**1.30:** 64

**1.31:**  $(1, 2) \cup (3, +\infty)$

**1.32:**  $x > 2$

**1.33:**  $[0, 1/2)$

**1.34:**  $\frac{\sqrt{2}}{4} < x < \frac{1}{2}$

**1.35:**  $\frac{a - b}{2}$

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## Chapter 2. Fundamentals of Complex Numbers

### Practice Questions:

**2.16:** 29

**2.17:** 240

**2.18:**  $-1$

**2.19:** 0

**2.20:** 9

**2.21(a):** Annular region between two concentric circles, both centered at the Origin, with radii 2 and 4, and the outer boundary is included, but the inner boundary is not included

**2.21(b):** The half plane whose  $y$ -coordinate is less than or equal to  $\frac{1}{2}$

**2.21(c):** all points  $(x,y)$  where  $x > 0$

**2.22:** 4

**2.23:** 54

**2.24:** Omitted

**2.25:**  $(x^4 + x^3 + x^2 + x + 1)(x^8 - x^7 + x^5 - x^4 + x^3 - x + 1)$

**2.26:**  $-1$

**2.27:** Omitted

**2.28:** Omitted

**2.29:**  $-\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$  and  $\text{cis} \frac{(2k+1)\pi}{13}$   
for  $0 \leq k \leq 5$  and  $7 \leq k \leq 12$ .

**2.30:**  $2^{5/4}$

**2.31:**  $300 - 50\pi$ .

**2.32:**  $-\frac{n}{2}$  and  $-\frac{n}{2} \cot \frac{\pi}{n}$

**2.33:**  $\frac{1}{3} \left( 2^n + 2 \cos \frac{n-2}{3} \pi \right)$

**2.34:**  $\frac{1}{3} \left( 2^n + 2 \cos \frac{n+2}{3} \pi \right)$

**2.35(a):**  $\frac{\sqrt{n}}{2^{n-1}}$

**2.35(b):**  $\frac{\sqrt{2n+1}}{2^n}$

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## Chapter 3. Solving Equations

### Practice Questions:

**3.16:**  $-1$

**3.17:**  $7$

**3.18:**  $8$

**3.19:**  $1001$

**3.20:**  $3$

**3.21:**  $7/2$

**3.22:**  $-3/2, 0, 3/2, 3$

**3.23:**  $1/2, 2, -2 \pm \sqrt{3}$

**3.24:**  $4, 5, 13$

**3.25:**  $10, -20.5$

**3.26:**  $2$

**3.27:**  $\pm\sqrt{3}$

**3.28:**  $a = 0$  or  $a > 25/4$

**3.29(a):** Omitted

**3.29(b):**  $0$  and  $7/12$

**3.30:**  $\frac{1 + \sqrt{13}}{2}$

**3.31:**  $\frac{\sqrt[3]{2}}{2}$

**3.32:**  $\frac{65}{63}$

**3.33:**  $1$

**3.34:**  $(0, 0), (4, 8), (4 \pm \sqrt{6}, 2 \pm 3\sqrt{6})$

**3.35:**  $7$  and  $38$

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## Chapter 4. Solving Inequalities

### Practice Questions:

**4.16:** 1

**4.17:**  $3(\sqrt{5} - 1)$

**4.18:** -6

**4.19:** 3.

**4.20:** 4

**4.21(a):**  $\sqrt[8]{8!} < \sqrt[9]{9!}$

**4.21(b):**  $37^{73} > 73!$

**4.22:**  $\left(-\frac{2+\sqrt{2}}{4}, 5+\frac{\sqrt{2}}{2}\right)$

**4.23:**  $(0, 2) \cup (256, +\infty)$

**4.24:**  $-\frac{\sqrt{10}}{5} \leq a \leq \frac{\sqrt{10}}{5}$ .

**4.25:** 47/48

**4.26:**  $\frac{5\sqrt{2}-7}{162}S^3$

**4.27:**  $1 \leq |\sin x| + |\cos x| \leq \sqrt{2}$

**4.28:**  $2\sqrt{5}$  and  $-\sqrt{15}$

**4.29:**  $\frac{8}{31} \leq a \leq \frac{72}{23}$

**4.30:**  $\frac{\sqrt{5}}{3}$

**4.31:**  $30^\circ$

**4.32:**  $(-\infty, -5) \cup (5, +\infty)$

**4.33:**  $\frac{n}{2n-1}$

**4.34:**  $1 + \sqrt{3}$

**4.35:** 25/16

**4.36:** 7/4

**4.37:**  $9\sqrt[4]{3}/8$

**4.38:**  $\frac{1+\sqrt{2}}{2}$

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## Chapter 5. Special Algebraic Techniques

### Practice Questions:

**5.16:** 2

**5.17:**  $x = 13$

**5.18:**  $5 \leq x \leq 10$

**5.19:**  $x = 1$

**5.20:**  $\frac{3 \pm \sqrt{5}}{2}$

**5.21:**  $(x, y) = (-26, 6)$  or  $(9, -29)$

**5.22:** 0

**5.23:**  $x = 1 \pm \sqrt{7}, 3 \pm \sqrt{15}$

**5.24:**  $a = -3, -1, 5, 15, 29$

**5.25:** 4035

**5.26:** 2.5

**5.27:** 2

**5.28:** 9

**5.29:** -1

**5.30:** 12