

Math Challenge III: Combinatorics

Answer Key

Areteem Institute

Chapter 1. Principles in Combinatorics

Practice Questions:

$$1.11: \binom{n}{6}$$

$$1.12: 34650 \text{ and } 7350$$

$$1.13: \frac{m-n+1}{m+1} \binom{m+n}{m}$$

$$1.14: \binom{n+1}{m}.$$

$$1.15: 3 \binom{n}{3} + n^3$$

$$1.16(a): \frac{(m+n+p)!}{m!n!p!}$$

$$1.16(b): \binom{m+p}{m} \cdot \binom{m+p+1}{n}$$

$$1.17(a): \binom{6}{2,2,2} = 90$$

$$1.17(b): \binom{6}{4,1,1} = 30$$

$$1.17(c): 3 \binom{6}{4,1,1} = 90$$

$$1.17(d): \binom{6}{3,2,1} = 60$$

$$1.17(e): 3! \cdot \binom{6}{3,2,1} = 360$$

$$1.18(a): \binom{6}{2,2,2} \cdot \frac{1}{3!} = 15$$

$$1.18(b): \binom{6}{4,1,1} \cdot \frac{1}{2!} = 15$$

$$1.18(c): \binom{6}{3,2,1} = 60$$

$$1.19(a): 10^n$$

$$1.19(b): 10 \times 11 \times \cdots \times (10+n-1) = {}_{10+n-1}P_n$$

$$1.20: 10395$$

Chapter 2. Combinatorics Problem Types

Practice Questions:

2.13: 78

2.19: 174

2.14: $\binom{9}{5} \binom{11}{6} = 58212$

2.20: 14

2.15: 420

2.21: 141

2.16: $\binom{n-1}{k-1} \binom{m-1}{k-2}$

2.22: 56

2.17: 998899

2.23: 90720

2.24: 210 and 455

2.18: $\binom{2005}{4}$

2.25: 7535

Chapter 3. Binomial Theorem

Practice Questions:

3.11: $\binom{2n}{n}$

3.12: $(1 + 3^{2000})2^{1999}$

3.13: Omitted

3.14: $n(n+1)2^{n-2}$

3.15: $\frac{1}{100}$

3.16: Omitted

3.17: $\frac{1}{2}(3^{2n+1} + 1)$

3.18: 71**3.19:** 1

3.20: $210x^6$ and $\frac{1}{32}$

Chapter 4. Bijection and Review Problems

Practice Questions:

$$4.10: \binom{50}{2} \binom{40}{2} = 955500$$

$$4.11: \binom{n+2}{2}$$

$$4.12: 2^{15} - 2 = 32766$$

$$4.13: \binom{30}{8} \binom{20}{8} 16!$$

$$4.14(a): 30! \cdot {}_{29}P_{20}$$

$$4.14(b): 29! \cdot {}_{30}P_{20}$$

$$4.15: 48! \cdot {}_{49}P_4$$

4.16: Omitted

$$4.17: 49$$

$$4.18: 732$$

$$4.19: 7^n$$

$$4.20: 3997$$

$$4.21: 2001$$

$$4.22: 43$$

$$4.23: 728$$

$$4.24: 768$$

$$4.25: 212 \text{ and } 695$$

$$4.26: 128$$

$$4.27: 28$$

$$4.28(a): 126$$

$$4.28(b): 24789$$

$$4.29: 174$$

$$4.30: 11$$

$$4.31: \binom{10}{7} \cdot 2^7 = 15360$$

$$4.32: 13440$$

$$4.33: 3 \binom{11}{1} \binom{10}{1} + 3! \cdot \binom{11}{1} \binom{10}{3} + 3 \binom{11}{1} \binom{10}{5} + 3 \binom{11}{3} \binom{8}{3} = 44286$$

$$4.34: 630$$

$$4.35: 36$$

$$4.36: \binom{19}{9} + \binom{10}{1} \binom{19}{8} + \binom{10}{2} \binom{19}{7} + \binom{10}{3} \binom{19}{6} = 6371498$$

$$4.37: 51601$$

$$4.38: 12504$$

$$4.39: n \cdot 2^{2n-2}$$

Chapter 5. Probability - The Classical Model

Practice Questions:

5.15: $1/3$

5.16: 0.18

5.17(a): $28/45$

5.17(b): $1/45$

5.17(c): $16/45$

5.17(d): $1/5$

5.18(a): $3/10$

5.18(b): $3/5$

5.19:
$$\frac{N+1}{M+N+1} \cdot \frac{n}{n+m} + \frac{N}{M+N+1} \cdot \frac{m}{n+m}$$

5.20: $53/99$

5.21: $3/5$

5.22: $20/21$

5.23(a): $\frac{3}{2}p - \frac{1}{2}p^2$

5.23(b): $\frac{2p}{p+1}$

5.24: Yes

Chapter 6. Geometrical Probability and More

Practice Questions:

6.10: $\left(1 - \frac{t_0}{T_2 - T_1}\right)^2$

6.11: $139/1152 \approx 0.121$

6.12: $\sqrt{3}/2$

6.13: $\frac{1}{2}$

6.14: $15/28$

6.15: $21/64$

6.16: $1/85$

6.17: $5/9$

6.18(a): $\Pr(X = k) = 0.45 \times (0.55)^{k-1}, k = 1, 2, \dots$

6.18(b): $11/31$

6.19(a): Geometric distribution,
 $\Pr(X = k) = (1/3) \cdot (2/3)^{k-1}$

6.19(b): $1/3$ for each of $Y = 1, 2, 3$

6.19(c): $8/27$

6.19(d): $38/81$

6.20(a): 0.32076

6.20(b): 0.243