

Math Challenge III: Number Theory

Answer Key

Areteem Institute

Chapter 1. Number Theory Review

Practice Questions:

1.11: Omitted

1.16: Omitted

1.12: $7 \times 13 \times 97 \times 163 \times 1201$

1.17: Omitted

1.13: (k, k, kc) where k and c are positive integers, in any order

1.18: Omitted

1.14: Omitted

1.19: Omitted

1.15: 105263157894736842

1.20: 142857

Chapter 2. Number Theory Practice

Practice Questions:

2.18: $p = 3, q = 2$

2.19: No

2.20: Omitted

2.21: 406

2.22: 259980

2.23: No

2.24: $2, 3, \dots, 30, 31$

2.25: No

2.26: No

2.27: Omitted

2.28: $n = 3$, and the sets are $\{1/2, 2/3, 6/7, 41/42\}$, $\{1/2, 2/3, 7/8, 23/24\}$, $\{1/2, 2/3, 8/9, 17/18\}$, $\{1/2, 2/3, 9/10, 14/15\}$, $\{1/2, 3/4, 4/5, 19/20\}$, and $\{1/2, 3/4, 5/6, 11/12\}$

2.29: Odd

2.30: No

2.31: 35964

2.32: Odd

Chapter 3. The Floor Function

Practice Questions:

3.14: No

3.21: 71

3.15: n

3.22: 42

3.16: $\sqrt[3]{4}$

3.23: $x \geq 2$

3.17: 3

3.24: 16

3.18: $n = k^2$ or $k^2 + k$ or $k^2 + 2k$ for positive integers k

3.25: 100800

3.19: Omitted

3.26: 1499

3.20: 7

3.27: Omitted

Chapter 4. Number Theory Functions

Practice Questions:

4.11(a): Yes

4.11(b): Yes

4.11(c): Omitted

4.12: 0

4.13: 1

4.14: $\prod_{\substack{p|n \\ p \text{ is prime}}} (-p)$

4.15: Odd

4.16: 43

4.17: $3^3 \times 7 \times 11 \times 13 \times 37 \times 101 \times 9901$

4.18: $a = 9, b = 2$

4.19: No

4.20: $x = 8, y = 0, z = 6$

4.21: Omitted

4.22: No

4.23: Omitted

4.24: 2016

4.25: 0

Chapter 5. Further Practice in Number Theory

Practice Questions:

5.19: 1

5.27: $(a + b + c + d)/4$

5.20: None exists

5.28: $\gcd(m + 1, n) = 1$

5.21: Omitted

5.29: 14

5.22: Omitted

5.30: No such integers exist

5.23: They are the same

5.31: Omitted

5.24: Omitted

5.32: 2^{1988}

5.25: $m = 30, n = 11$

5.33: Yes

5.26: $n = 4, n = 8, n \geq 10$