Math Challenge III: Number Theory
Answer Key
Areteem Institute
Chapter 1. Number Theory Review

Practice Questions:

1.11: Omitted

1.12: $7 \times 13 \times 97 \times 163 \times 1201$

1.13: $(k, k, kc)$ where $k$ and $c$ are positive integers, in any order

1.14: Omitted

1.15: 105263157894736842

1.16: Omitted

1.17: Omitted

1.18: Omitted

1.19: Omitted

1.20: 142857
Chapter 2. Number Theory Practice

Practice Questions:

2.18: \( p = 3, q = 2 \)

2.19: No

2.20: Omitted

2.21: 406

2.22: 259980

2.23: No

2.24: 2, 3, \ldots, 30, 31

2.25: No

2.26: No

2.27: Omitted

2.28: \( n = 3, \) and the sets are \{1/2, 2/3, 6/7, 41/42\}, \{1/2, 2/3, 7/8, 23/24\}, \{1/2, 2/3, 8/9, 17/18\}, \{1/2, 2/3, 9/10, 14/15\}, \{1/2, 3/4, 4/5, 19/20\}, and \{1/2, 3/4, 5/6, 11/12\}

2.29: Odd

2.30: No

2.31: 35964

2.32: Odd
Chapter 3. The Floor Function

Practice Questions:

3.14: No

3.15: n

3.16: $\sqrt{4}$

3.17: 3

3.18: $n = k^2$ or $k^2 + k$ or $k^2 + 2k$ for positive integers $k$

3.19: Omitted

3.20: 7

3.21: 71

3.22: 42

3.23: $x \geq 2$

3.24: 16

3.25: 100800

3.26: 1499

3.27: Omitted

3.28: Omitted
Chapter 4. Number Theory Functions

Practice Questions:

4.11(a): Yes
4.11(b): Yes
4.11(c): Omitted
4.12: 0
4.13: 1
4.14: \( \prod_{p|n} (-p) \) if \( p \) is prime
4.15: Odd
4.16: 43

4.17: \( 3^3 \times 7 \times 11 \times 13 \times 37 \times 101 \times 9901 \)
4.18: \( a = 9, b = 2 \)
4.19: No
4.20: \( x = 8, y = 0, z = 6 \)
4.21: Omitted
4.22: No
4.23: Omitted
4.24: 2016
4.25: 0
Chapter 5. Further Practice in Number Theory

Practice Questions:

5.19: 1  
5.20: None exists  
5.21: Omitted  
5.22: Omitted  
5.23: They are the same  
5.24: Omitted  
5.25: \( m = 30, n = 11 \)  
5.26: \( n = 4, n = 8, n \geq 10 \)  
5.27: \( (a + b + c + d)/4 \)  
5.28: \( \gcd(m + 1, n) = 1 \)  
5.29: 14  
5.30: No such integers exist  
5.31: Omitted  
5.32: \( 2^{1988} \)  
5.33: Yes